4.7 Identity and Inverse Matrices

Identity Matrix--square matrix that when multiplied by another matrix, it equals that same matrix.

$$A \cdot I = A \qquad I \cdot A = A$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse matrices--are 2 square matrices whose product is the identity

A^{-1} -- "A inverse"

$$A \cdot A^{-1} = I$$

$$A^{-1} \cdot A = I$$

Are they inverses?

yes $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

X $\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ Y $= \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$X \cdot Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $YX = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Are they inverses? NO

$P = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$ $Q = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

$PQ = \begin{bmatrix} 1 & -13 \end{bmatrix}$

Finding the inverse.

If D = 0, there is no inverse.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \frac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Find the inverse.

$$A = \begin{bmatrix} 6 & 4 \\ -1 & 3 \end{bmatrix} \quad -4 \quad 18$$

$$A^{-1} = \frac{1}{22}\begin{bmatrix} 3 & -4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} \frac{3}{22} & -\frac{2}{11} \\ \frac{1}{22} & \frac{3}{11} \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 \\ 8 & -2 \end{bmatrix} \quad 0 \quad 2$$

$$B^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 0 \\ -8 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -4 & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} -4 & 6 \\ -2 & 3 \end{bmatrix} \quad D = 0 \quad \text{No Inverse}$$

$$B = \begin{bmatrix} -1 & 0 \\ 8 & -2 \end{bmatrix} \qquad B^{-1} = \begin{bmatrix} -1 & 0 \\ -4 & -\frac{1}{2} \end{bmatrix}$$

$$B \cdot B^{-1} = \begin{bmatrix} 1,1 & 1,2 \\ 2,1 & 2,2 \end{bmatrix} \qquad 2\times2 \quad 2\times2$$

$$1,1 = -1(-1) + 0(-4) = 1$$
$$1,2 = -1(0) + 0\left(-\tfrac{1}{2}\right) = 0$$
$$2,1 = 8(-1) + -2(-4) = 0$$
$$2,2 = 8(0) \quad -2\left(-\tfrac{1}{2}\right) = 1$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Cryptography

_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

13|38|24|49|44|107|19|57|22|53|17|39

$$B = \begin{bmatrix} 13 & 38 \\ 24 & 49 \\ 44 & 107 \\ \vdots & \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$B \cdot A^{-1}$ = Message

$6\times2 \quad 2\times2$

_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

PARABOLA

$$B=\begin{bmatrix}16 & 5\\ 18 & 1\\ 2 & 15\\ 12 & 1\end{bmatrix} \qquad A=\begin{bmatrix}12\\ 13\end{bmatrix} \qquad B\cdot A=\begin{bmatrix}17 & 35\\ 19 & 39\\ 17 & 49\\ 13 & 27\end{bmatrix}$$

_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

14|29|28|64|20|60|24|67|20|40|29|72|13|39|29|72|5|11|25|50

MATHTEST ON Ch 4 but is MONDAY

HW
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